

BSFT and Closed Strings

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MB, I. Sachs, S. Shatashvili, 2004
MB, I. Sachs, 2006
MB, I. Sachs, 2008
MB, 2008

Open strings vs. closed strings

- ▶ **S-matrix:**
closed string poles in open string scattering amplitudes
unitarity in OSFT?
- ▶ **2d string theory (Liouville):** explicit duality
- ▶ **tachyon condensation:** open string completeness conjecture
- ▶ **topological string theory:** e.g. Hochschild complex

Open strings vs. closed strings

- ▶ **BCFT:** open and closed string moduli
- ▶ open moduli space changes significantly under closed string deformations
- ▶ dramatic effects on D-branes

[MB, Brunner, Gaberdiel, 2007]

[MB, Wood, 2008]

- ▶ how do closed string deformations appear in OSFT?
- ▶ background (in)dependence?

Outline

Introduction

BSFT approach

BSFT in a nutshell

Factorization

BSFT on WZW

Factorization for boundary WZW

BSFT action

Curved background, an example

SU(2)

Tachyon condensation

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BSFT in a nutshell

- ▶ $S = \int_{\Sigma} L(G_{\mu\nu}; B_{\mu\nu}; X^{\mu}) + \int_{\partial\Sigma} b(t^I; X^{\mu})$
 - ▶ L defines a CFT as 'background' with massless fields as moduli
 - ▶ b is a boundary term with a 'complete' set of couplings t^I
 - ▶ and vertex operators $V_I = \left. \frac{\partial b(t^J)}{\partial t^I} \right|_{t^J=0}$
 - ▶ b breaks conformal invariance (on the boundary only)
- ▶ classical solutions of SFT: ghost + matter CFT
- ▶ idea: consider the space of all *boundary deformations* \mathcal{O} (ghost number 1)

$$S = S_{\text{matter}} + S_{\text{ghosts}} + \oint b_{-1} \mathcal{O}$$

$$b_{-1} = \oint_{C \rightarrow \partial D} b, \quad Q_B = \oint_{C \rightarrow \partial D} J_B$$

BSFT in a nutshell

- ▶ construct action via BV:
use master equation (choose anti-bracket) + classical solutions
- ▶ SFT action (*form on coupling space*):

$$d\mathcal{S} = \left\langle \int d\mathcal{O} \left\{ Q_B, \int \mathcal{O} \right\} \right\rangle$$

[Witten]

- ▶ matter/ghost decoupling: $\mathcal{O} = c\mathcal{V} = ct^l\mathcal{V}_l$

$$\mathcal{S} = \left(1 - \beta^l(t) \frac{\partial}{\partial t^l} \right) \mathcal{Z}(t)$$

[Shatashvili]

- ▶ generating functional $\mathcal{Z}(t^l) = \int DX e^{-S[X]}$

Space of boundary couplings

- ▶ consider flat background $\mathbb{R}^{1,25}$ with string field X
derivative expansion:

$$\mathcal{V}(X) = T(X) + A_\mu(X)\dot{X}^\mu + \text{massive modes}$$

- ▶ \mathcal{S} is a functional of *infinitely many* modes (*renormalizability!*)
- ▶ these might be expansions of non-local interactions [*Li, Witten*]

$\oint X(\sigma)u(\sigma, \sigma')X(\sigma')$ can be expanded in
derivatives $\sum_n \oint X t_n \partial_\sigma^{(n)} X$ with $u^n = \sum_m (in)^m t_m$

- ▶ such 'collectively excited' couplings appear naturally and get interpretation from closed string sector

Background dependence?

$$\mathcal{M} = (G_{\mu\nu}, B_{\mu\nu}) \quad \longleftrightarrow \quad \mathcal{M}' = (G'_{\mu\nu}, B'_{\mu\nu})$$

- ▶ OS spectrum may change
- ▶ OS β -equations may change
- ▶ OS conformal point shifted $t_* \rightarrow t'_*$
- ▶ BSFT action $\mathcal{S} = (1 - \beta^I \partial_I) \mathcal{L}$ defined pointwise

NB: reminiscent of the situation in open-closed moduli spaces,
Picard-Fuchs equation systems

Background independence!

$$\mathcal{M} = (G_{\mu\nu}, B_{\mu\nu}) \quad \longleftrightarrow \quad \mathcal{M}' = (G'_{\mu\nu}, B'_{\mu\nu})$$

$$\mathcal{S}_{\mathcal{M}}(t^l) \quad \longleftrightarrow \quad \mathcal{S}_{\mathcal{M}'}(t^l + \Delta t^l; \tau^l)$$

- ▶ new open string background $t'_* = t_* + \Delta t$
- ▶ the original couplings t^l still describe the open strings
- ▶ surprise: no need to integrate out closed strings, due to a factorization property; no higher order α' terms in the boundary interaction term b

$$\mathcal{L}_{\mathcal{M}} = \mathcal{L}_0 \mathcal{L}_{\mathcal{M}'}$$

- ▶ but: generically non-local couplings τ^l appear

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Re-formulation of BSFT

splitting of fields

- ▶ how to parametrize space of boundary-deformed 2d CFTs?
- ▶ $\{\text{space of boundary deformations}\} \longleftrightarrow \{\text{space of functions } f \text{ on } S^1\}$

(also parametrizes all boundary conditions, after integration)

- ▶ unique decomposition: $X = X_0 + X_b$
where $X_0| = x_0$ (D0) and $\Delta X_b = 0$ (harmonic)
- ▶ thus $X_b = X_b[f]$ with $X_b(\tau, \sigma)| = f(\sigma)$
- ▶ $S(X) = \int d^2z \partial X \bar{\partial} X = S(X_0) + S(X_b) + W(X_0, X_b)$
- ▶ with $S(X_b) = \oint d\sigma d\sigma' f(\sigma) H(\sigma - \sigma') f(\sigma')$
- ▶ and $H(\sigma) = \sum_n \oint d\sigma |n| e^{in\sigma}$

Re-formulation of BSFT

more on the boundary action

- ▶ boundary data $f(\sigma)$ can be decomposed by holomorphic/anti-holomorphic extendibility on the disk
- ▶ $f(\sigma) = f_+(\sigma) + f_-(\sigma)$
 $\longrightarrow f_+(z) + f_-(\bar{z}) = f(z, \bar{z}) = X_b(z, \bar{z})$
- ▶ action of H : $Hf = \partial_\sigma f_+ - \partial_\sigma f_- = \partial_n X_b$
- ▶ boundary action: $\oint f_+ df_- = \oint X_b \partial_n X_b$
observation: 2-cocycle

Re-formulation of BSFT

partition function

- ▶ replace $\mathcal{Z}[t]$ by $\mathcal{Z}[f]$
 $\mathcal{Z}[f]$ – partition function with fixed boundary conditions f :
 $\mathcal{Z}[t] = \int Df e^{-I[t]} \mathcal{Z}[f]$
- ▶ path-integral factorizes:

$$\mathcal{Z}[f] = \int_{D0 \text{ b.c.}} DX_0 e^{-S[X_0]} \times e^{-\tilde{S}[f]} = Z_0 e^{-\tilde{S}[f]}$$

- ▶ no ‘interaction’ between ‘bulk part’ and ‘boundary part’ of X
- ▶ X_0 has not been ‘integrated out’! I.e. arbitrary boundary interactions allowed

Re-formulation of BSFT

partition function

- ▶ factorization allows to re-define BSFT-action
(requires choice of measure for f -path-integral)

$$\mathcal{S} \longrightarrow \mathcal{S}' = \mathcal{S} / \mathcal{L}_0$$

- ▶ everything formulated in terms of 'boundary fields' f
- ▶ $\mathcal{S}' = \mathcal{S} / \mathcal{L}_0$ is well-defined
- ▶ change in closed string background can be studied in BSFT
- ▶ how generic is this factorization property?

Simple examples

Starting from the free boson $\mathcal{M} = (G_{\mu\nu} = \eta_{\mu\nu}, B_{\mu\nu} = 0, R)$

- ▶ a shift in the Kalb-Ramond field yields

$$\delta B_{\mu\nu} \oint X_b^\mu dX_b^\nu \longrightarrow \text{gauge field}$$

- ▶ a shift in the compactification radius R yields

$$\frac{\delta R}{R} \oint X_b \partial_n X_b$$

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Remarks on WZW, closed string only

- ▶ model for non-trivial (flat) closed string background
- ▶ $S_{WZW}(g) = \kappa L(g) + \kappa \Gamma(g)$

$$L = \text{Tr} \int g^{-1} \partial g g^{-1} \bar{\partial} g$$

$$\Gamma = \text{Tr} \int w_3$$

$$w_3 = (dg g^{-1})^{\wedge 3}$$

- ▶ for w_3 extension of g to interior of closed worldsheet necessary
- ▶ no ambiguities in partition function

Remarks on boundary WZW

- ▶ for simplicity assume $H^3 = 0$
- ▶ w_3 can be pulled to the boundary, $w_3 = dw_2$
- ▶ ambiguity: w_2 defined up to closed form $d\beta$
- ▶ thus for the action one needs to specify a one-form β :

$$\Gamma^\beta = \Gamma + \oint \beta$$

- ▶ β lives on the boundary only
- ▶ some assumptions needed to fix β (below)

Factorization for boundary WZW

- ▶ Splitting of the fields:

$$\begin{aligned} g &= g_0 k && \text{in analogy to } X = X_0 + X_b \\ k &= h(z)\bar{h}(\bar{z}) && \text{in analogy to } X_b = f_+(z) + f_-(\bar{z}) \end{aligned}$$

- ▶ g_0 satisfies D0-boundary conditions, $g_0| = \text{const}$
 k satisfies classical EOM, $\partial k k^{-1} = 0 = k^{-1} \bar{\partial} k$
- ▶ $S(g_0 k)$ determined by Polyakov-Wiegmann formula, but needs extension for boundary WZW

Boundary Polyakov-Wiegmann

- ▶ only Γ may be problematic.
for closed string: $w_3(g_1 g_2) = w_3(g_1) + w_3(g_2) + G(g_1, g_2)$
- ▶ for open string one can show, that
 $w_3(g_1 g_2) - w_3(g_1) - w_3(g_2) - G(g_1, g_2)$ is a 2-cocycle on LG
- ▶ and its integral over the worldsheet depends only on its values on the boundary and is independent of the choice of extension
- ▶ for $g = g_0 k$ this integral vanishes
- ▶ finally,

$$S_{WZW}(g) = S_{WZW}(g_0) + L(k) + \Gamma^\beta(k) + W(g_0, k)$$

$$\text{with } W(g_0, k) = \text{Tr} \int g_0^{-1} \bar{\partial} g_0 \partial k k^{-1}$$

Factorization

$$S_{WZW}(g) = S_{WZW}(g_0) + L(k) + \Gamma^\beta(k) + W(g_0, k)$$

- ▶ a priori no decoupling of g_0 and k , due to presence of W
- ▶ but still, the partition function factorizes (*main technical result*):

$$\left\langle \left(\text{Tr} \int g_0^{-1} \bar{\partial} g_0 \partial k k^{-1} \right)^n \right\rangle_{g_0} = 0$$

- ▶ thus W **does not** contribute
- ▶ $Z[k] = Z_0 e^{-\tilde{S}[k]}$ is still true

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The boundary action

$$S(k) = L(k) + \Gamma^\beta(k)$$

- ▶ explicit expression wanted
- ▶ using the Ansatz $k(z, \bar{z}) = h(z)\bar{h}(\bar{z})$ and boundary Polyakov-Wiegmann, we get

$$\Gamma^\beta(k) = -L(k) + \int \alpha(h, \bar{h})$$

where α is a 2-cocycle

- ▶ therefore factorization gives

$$Z[k] = Z_0 e^{-\int \alpha(h, \bar{h}) - \beta}$$

- ▶ we cannot calculate the boundary action, but we can make a guess based on some basic properties

The boundary action

the chosen action must ...

- ▶ ... lead to the same algebraic boundary conditions as the 'full' WZW model
- ▶ ... obey the cocycle conditions
- ▶ ... give the correct large-radius limit
- ▶ our Ansatz:

$$\tilde{S} = L(k) = \int k^{-1} \partial k k^{-1} \bar{\partial} k = \int h^{-1} \partial h \bar{\partial} \bar{h} h^{-1}$$

(can be written as a surface integral)

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(can be written as a surface integral)

... SL(2), SU(2), Nappi-Witten ...

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WZW on SU(2)

- ▶ symmetries
- ▶ maximally symmetric D-branes known \leftrightarrow conjugacy classes, discrete
- ▶ parametrization:

$$k = e^{i\lambda X^\mu T_\mu}$$

$\lambda = \kappa^{-\frac{1}{2}}$, expansion parameter

T – generators of SU(2)

large radius limit: $\lambda \rightarrow 0 \quad \Leftrightarrow \quad \kappa \rightarrow \infty$

- ▶ in the same way:

$$h(z) = e^{i\lambda f_+^\mu T_\mu} \quad \bar{h}(\bar{z}) = e^{i\lambda f_-^\mu T_\mu}$$

Coordinates

Relation between flat and curved coordinates

$$X^\mu = f_+^\mu + f_-^\mu - \lambda \varepsilon_{\mu\nu\kappa} f_+^\nu f_-^\kappa + \mathcal{O}(\lambda^2)$$

- ▶ continuation: $f_+ \rightarrow f(z)$ $f_- \rightarrow \bar{f}(\bar{z})$
- ▶ technical issue: enforces $SU(2)^\mathbb{C}$

Non-local deformations

- ▶ definition of a path-integral:

$$\int [\delta k k^{-1}] e^{-\frac{1}{(i\lambda)^2} \int \partial k k^{-1} \bar{\partial} k k^{-1}}$$

- ▶ express everything in f, \bar{f}
- ▶ action:

$$S = s \sum_{m=1}^{\infty} m f_m \bar{f}_m - \frac{\lambda}{2} \{ V_\alpha - \bar{V}_\alpha + \lambda V_\beta + \lambda V_\gamma + \lambda \bar{V}_\gamma \}$$

$$V_\alpha = \sum (c-b) \delta_{a,c+b} \varepsilon_{\mu\nu\lambda} f_c^\mu f_b^\nu \bar{f}_a^\lambda$$

$$V_\beta = \sum \frac{(c-b)(a-d)}{a+d} \delta_{c+b,a+d} f_c^\mu f_b^\nu \bar{f}_{a\mu} \bar{f}_{d\nu}$$

$$V_\gamma = \frac{2}{3} \sum (a-b-d) \delta_{c,a+b+d} f_c^\mu \bar{f}_{a\mu} \bar{f}_b^\nu \bar{f}_{d\nu}$$

Non-local deformations

- ▶ contribution from measure:

$$S_{\text{measure}} = 4\lambda^2 \sum f_n \bar{f}_n$$

- ▶ include quadratic tachyon

$$S_{\text{tachyon}} = \oint T(X) = a + u \sum f_n \bar{f}_n$$

Non-local deformations

- ▶ contribution from measure:

$$S_{\text{measure}} = 4\lambda^2 \sum f_n \bar{f}_n$$

(tachyon forced into existence!)

- ▶ include quadratic tachyon

$$S_{\text{tachyon}} = \oint T(X) = a + u \sum f_n \bar{f}_n$$

RG flow

- λ → non-local deformation (keep symmetry)
 a, u → control tachyon condensation

β -functions

$$\beta_a = -a - u$$

$$\beta_u = -u(1 - 8\lambda^2)$$

$$\beta_\lambda = -\frac{47}{6}\lambda^3$$

- ▶ λ increases
- ▶ tachyon condensation is modified
- ▶ obvious endpoint: $a = \infty$, $u = \infty$ (but there is more...)

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Condensation of the 3-brane

- ▶ non-vanishing tachyon makes 3-brane unstable
- ▶ can we identify a process $D3 \rightarrow D2$?
- ▶ assume a tachyon interaction

$$\oint \rho (X^2 - c^2)^2 = \rho c^4 - 4\rho c^2 \sum f_n \bar{f}_n + \mathcal{O}(f^4)$$

- ▶ ρ, c^2 are the 'natural couplings', in which a condensation to spherical brane is described
- ▶ re-write the β -functions:

$$\beta_{c^2} = 4 - 8\lambda^2$$

$$\beta_{\rho} = -\frac{\rho}{c^2} (c^2 - 16c^2\lambda^2 + 4)$$

Condensation of the 3-brane

- ▶ for a 2-brane solution we would expect finite c^2 and running ρ (λ fixed)
- ▶ such a solution is

$$c^2 = \frac{1}{2}\lambda^{-2}$$

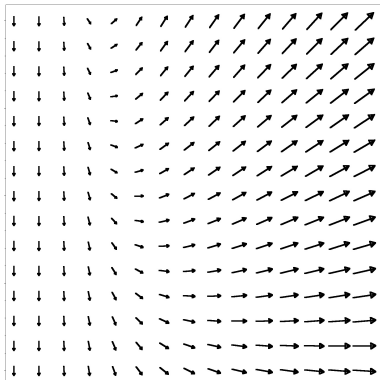
$$\beta_{c^2} = 0$$

$$\beta_\rho = -\rho(1 - 8\lambda^2)$$

- ▶ this tells us: ρ will increase
- ▶ geometry: determined by λ (curvature)
- ▶ $\lambda \rightarrow 0$ gives a flat brane (*flat space limit*)

λ - c^2 -flow

- ▶ running of c^2 is strongly modified by the presence of λ



Verification of the solution

- ▶ Ansatz for D2-solution was a ‘guess’
- ▶ independent verification:
 - ▶ insert constraint $X^2 = c^2$ into action
 - ▶ verify conformality
- ▶ complicated \rightarrow look for stability check

$$A_1 = \frac{1}{2} \sum_{a=1} \sum_{d=2} \sum_{e=1}^{d-1} (d-a) \epsilon_{\alpha\beta} f_a^\alpha \bar{f}_{a+d}^\beta f_{e\gamma} \bar{f}_{d-e}^\gamma$$

$$\bar{A}_1 = A_1^*$$

$$A_2 = \sum_{a=1} \sum_{c=1} \sum_{d=2} (d-a) \epsilon_{\alpha\beta} f_a^\alpha \bar{f}_{a+d}^\beta f_{c+d\gamma} \bar{f}_c^\gamma$$

$$\bar{A}_2 = A_2^*$$

$$A_3 = \frac{1}{4} \sum_{a=1} \sum_{b=1} \sum_{g=1}^{a+b-1} (a-b) \epsilon_{\alpha\beta} f_a^\alpha f_b^\beta \bar{f}_{g\gamma} \bar{f}_{a+b-g}^\gamma$$

$$\bar{A}_3 = A_3^*$$

$$A_4 = \frac{1}{2} \sum_{a=1} \sum_{b=1} \sum_{c=1} (a-b) \epsilon_{\alpha\beta} f_a^\alpha f_b^\beta f_{c\gamma} \bar{f}_{a+b+c}^\gamma$$

$$\bar{A}_4 = A_4^*$$

$$B_1 = \frac{1}{4} \sum_{m=2} \sum_{a=1}^{m-1} \sum_{b=1}^{m-1} m f_{a\alpha} f_{m-a}^\alpha \bar{f}_{b\beta} \bar{f}_{m-b}^\beta$$

$$B_2 = \sum_{m=2} \sum_{a=1}^{m-1} \sum_{b=1}^{m-1} m \bar{f}_{a\alpha} f_{m-a}^\alpha f_{b\beta} \bar{f}_{m-b}^\beta$$

$$B_3 = \frac{1}{2} \sum_{m=2} \sum_{a=1}^{m-1} \sum_{b=1}^{m-1} m f_{a\alpha} f_{m-a}^\alpha f_{b\beta} \bar{f}_{m-b}^\beta$$

$$C_1 = - \sum_{a,b,c,d=1} \frac{(a-b)(c-d)}{c+d} f_a^\alpha \bar{f}_{c\alpha} f_b^\beta \bar{f}_{d\beta} \delta_{a+b,c+d}$$

$$C_2 = \frac{1}{3} \sum_{a,b,c,d=1} (c-a-b) \bar{f}_a^\alpha \bar{f}_{b\alpha} \bar{f}_{c\beta} f_{a+b+c}^\beta$$

$$\bar{C}_2 = C_2^*$$

Stability of the 2-brane

- ▶ consider case with vanishing tachyon
- ▶ will the tachyon be excited at higher loops?
- ▶ the tachyon counter-term:

$$\Sigma^{(2)}(p=0) = [\Lambda \ln \Lambda + (\gamma - 1)\Lambda] \left\{ \frac{4\lambda^2}{c^2 s^3} - \frac{8}{3} \frac{\lambda^2}{s^2 c^2} - \frac{38}{sc^4} \right\} \\ + \ln \Lambda \{ \text{terms} \propto u \} + \mathcal{O}(\lambda^4)$$

- ▶ all divergences vanish under a single condition:

$$c^2 = \alpha(s)\lambda^{-2}$$

- ▶ no 'fine-tuning' necessary

Interpretation

On the three-brane:

- ▶ λ excites infinitely many massive couplings
- ▶ affects stability of D-branes
- ▶ tachyon condensation process initiated and modified
- ▶ qualitatively new RG flows
- ▶ λ is running, c^2 can be kept finite, ρ is running & condensating
- ▶ breakdown of predictability \rightarrow 2-brane Ansatz

On the two-brane:

- ▶ a priori tachyonic instability
- ▶ instability removed by essentially same condition on c^2 as on three-brane
- ▶ tachyon can be set to zero without fine-tuning
- ▶ indicates that 2-brane is conformal
- ▶ λ really seems to interpolate between flat and curved space \rightarrow supports interpretation of λ as closed string perturbation

Perspectives

- ▶ higher order β -functions, more examples, better methods
- ▶ better control over flow
- ▶ other WZW-branes, potential
- ▶ backgrounds other than WZW
- ▶ supersymmetry
- ▶ systematic investigation of open-closed moduli spaces, better statements on background independence
- ▶ boundary state formalism, possibly access to closed string vacuum
- ▶ connections to T-folds? Defect CFTs?

THANK YOU