Unfolded Description of AdS_4 Black Hole

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Plan

- **Introduction.**

  Motivation. Some important black hole properties. Unfolded formulation of dynamical systems.

- AdS_4 **space-time. Unfolded equations.**

- AdS_4 **black hole as the deformation of vacuum system.**

- **Integrating flow. Explicit coordinate independent BH metrics.**

- **Conclusion.**

Motivation

Gravity s=2 $\Longrightarrow$ Super-gravity $\Longrightarrow$ Higher spin gauge theory (HS) $s = 0, 1/2, 1, \ldots, \infty$ **(Vasiliev)**

HS gauge theory:

Consistent theory of interacting massless fields $s = 0, 1/2, \ldots \infty$ **in** AdS **space-time**

Current status:

Formulated at the level of equations of motion for all spins in $d = 4$ **Bosonic version is known in any** d

String theory $\rightarrow$ **unbroken phase** $\rightarrow$ **HS theory** **(???)**

Lack of:

Action principle, quantization.

GR black holes $\longrightarrow$ SUGRA black holes $\longrightarrow$ HS black holes ???

Obstacles:

1. HS does not have decoupled spin-2 sector $\rightarrow$ all higher spins involved in the equations of motion.

2. The interval $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ is not gauge invariant quantity in higher spin algebra.

3. Considerable technical difficulties – the equations are essentially non-local involving space-time derivatives of all orders.

Perturbative analysis available

HS theory $\longrightarrow$ 0-th order vacua AdS $\longrightarrow$ 1-st order free field Fronsdal equations $\longrightarrow$ 2-nd order interactions $\longrightarrow \ldots$

Classical black hole properties

Ex. $d=4$ Kerr Solution

1. $g_{\mu\nu} = \eta_{\mu\nu}(x) + M h_{\mu\nu}(x)$ – no $O(M^2)$ terms $\Longrightarrow$ Einstein equations reduce to **free** $s=2$ Pauli-Fierz eqs.

$$\Box h_{\mu\nu} - \partial_\mu \partial_\lambda h^\lambda{}_\nu - \partial_\nu \partial_\lambda h^\lambda{}_\mu = 0 \qquad (h_\mu{}^\mu = 0)$$

2. $h_{\mu\nu} = \frac{1}{U(x)} k_\mu(x) k_\nu(x)$ – factorized form. k^μ – Kerr-Schild vector

$$k_\mu k^\mu = 0\,, \qquad k^\mu D_\mu k_\nu = k^\mu \partial_\mu k_\nu = 0$$

3. BH provides Fronsdal fields $\phi_{\mu_1 \dots \mu_s} = \frac{M}{U} k_{\mu_1} \dots k_{\mu_s}$

$\mathbf{s=0}$ $\Longrightarrow$ Klein-Gordon $\qquad \Box\phi = 0$

$\mathbf{s=1}$ $\Longrightarrow$ Maxwell $\qquad \phi_\mu - \partial_\lambda \partial_\mu \phi^\lambda = 0$

$\mathbf{s=2}$ $\Longrightarrow$ Pauli-Fierz $\qquad \Box\phi_{\mu\nu} - 2\partial_\lambda \partial_{(\mu}\phi_{\nu)}{}^\lambda = 0$

$\mathbf{s=s}$ $\Longrightarrow$ Fronsdal $\qquad \Box\phi_{\mu_1 \dots \phi_{\mu_s}} - s\partial_\lambda \partial_{(\mu_1}\phi^\lambda{}_{\mu_2 \dots \mu_s)} = 0$

4. Kerr-Schild presentation is also valid in AdS

$$g_{\mu\nu} = \eta^{AdS}_{\mu\nu}(x) + \frac{M}{U} k_\mu k_\nu \,, \quad k^\mu k_\mu = 0 \,, \quad k^\mu \mathcal{D}_\mu k_\nu = k^\mu D_\mu k_\nu = 0$$

Just as well,

$$\phi_{\mu_1 \ldots \mu_s} = \frac{M}{U} k_{\mu_1} \ldots k_{\mu_s}$$

satisfies free massless spin-s equations (**Metsaev**) in AdS_4

$$\Box \phi_{\mu_1 \ldots \mu_s} - s D_\lambda D_{(\mu_1} \phi^\lambda{}_{\mu_2 \ldots \mu_s)} = -2(s-1)(s+1)\lambda^2 \phi_{\mu_1 \ldots \mu_s}$$

$\phi_{\mu_1 \ldots \mu_s}(x)$ − **Black hole massless fields**

Program for HS black holes

vacuum AdS_4	$\longrightarrow$	black hole massless fields $\phi_{\mu_1 \ldots \mu_s}(x)$	$\longrightarrow$	HS corrections	$\longrightarrow \ \ldots$

Unfolded formulation

- First order coordinate independent differential equations (differential forms formalism)

- Assumes additional fields (generally infinitely many) that parameterize all on-shell derivatives of physical fields

Example: free massless scalar in Minkowski space-time $\Box\phi(x) = 0$

Unfolding $\rightarrow \varphi(x), \quad \varphi_\mu = \partial_\mu \varphi, \quad \varphi_{\mu\nu} = \partial_\mu \varphi_\nu, \ldots \quad \varphi_{\mu_1 \ldots \mu_n} = \partial_{\mu_1} \varphi_{\mu_2 \ldots \mu_n},$

$\ldots$

Set of fields: $\phi, \quad \phi_\mu, \ldots \quad \varphi_{\mu_1 \ldots \mu_n}, \ldots$

Consistency condition: $\varphi_{\mu_1 \ldots \mu_n}$ – symmetric

Equations of motion: ${\varphi^\mu}_{\mu\mu_3 \ldots \mu_n} = 0$

To perform perturbative analysis of black holes in HS theory one has to have explicit expressions for all AdS_4 derivatives of AdS_4-Kerr black hole fields (vierbein, curvature). Therefore, one needs AdS_4 covariant description of classical black hole to proceed.

Strategy

1. **Find in pure AdS_4 space-time objects relevant to black hole such as Kerr-Schild vectors and blocks of BH curvature**

2. **Find appropriate deformation of AdS_4 equations that leads to BH**

3. **Find integrating flow with respect to deformation parameters mapping one system to another. Try to integrate flow equations with AdS_4 initial data**

Cartan formalism

Instead of $g_{\mu\nu} \to$ Lorentz connection 1-form $\Omega_{[ab]} = \Omega_{[ab],\mu} dx^\mu$ and vierbein 1-form $h_a = h_{a,\mu} dx^\mu$, $a, b = 1, \ldots, 4$

$$R_{ab} = d\Omega_{ab} + \Omega_a{}^c \wedge \Omega_{cb} \qquad \text{Riemann 2-form}$$

$$R_a = dh_a + \Omega_a{}^b \wedge h_b = 0 \qquad \text{torsion 2-form}$$

$$g_{\mu\nu} = h_{a,\mu} h_{b,\nu} \eta^{ab}$$

Two-component spinor notation

$V^a \to V^{\alpha\dot{\alpha}}$, $\quad \alpha, \dot{\alpha} = 1, 2$, $\quad$ indicies raised and lowered with $\quad \epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}$

$$F_{ab} = -F_{ba} \to (F_{\alpha\alpha}, \bar{F}_{\dot{\alpha}\dot{\alpha}}), \quad C^{\boxplus}_{abcd} \text{(Weyl tensor)} \to (C_{\alpha(4)}, \bar{C}_{\dot{\alpha}(4)})$$

$$\mathrm{R}_{ab} \quad \text{(Ricci)} \to \Phi_{\alpha\dot{\alpha}\alpha\dot{\alpha}}$$

AdS_4 space-time (A)

$$ds^2 = -du^2 - dv^2 + dx^2 + dy^2 + dz^2\,, \qquad -u^2 - v^2 + x^2 + y^2 + z^2 = \lambda^{-2}$$

Isometries: $o(3,2) \Longrightarrow$ 10 Killing vectors. Let V^a be an AdS_4 Killing vector:

$$D_a V_b + D_b V_a = 0\,, \qquad \kappa_{ab} = D_a V_b = -\kappa_{ba}$$

(Ricci identity: $D_a D_b V_c = R^d{}_{abc} V_d$**)**

$$DV_a = \kappa_{ab} h^b\,, \quad D\kappa_{ab} = \lambda^2 (V_a h_b - V_b h_a) \longleftarrow$$ **unfolded equations**

$$d\Omega_{ab} + \Omega_a{}^c \wedge \Omega_{cb} = \lambda^2 h_a \wedge h_b\,, \quad dh_a + \Omega_a{}^b \wedge h_b = 0 \longleftarrow$$ **consistency,** AdS_4

Spinor form for AdS_4 equations:

$$DV_{\alpha\dot\alpha} = \frac{1}{2} h^{\gamma}{}_{\dot\alpha}\kappa_{\gamma\alpha} + \frac{1}{2} h_{\alpha}{}^{\dot\gamma}\bar\kappa_{\dot\alpha\dot\gamma}$$

$$D\kappa_{\alpha\alpha} = \lambda^2 h_{\alpha}{}^{\dot\gamma} V_{\alpha\dot\gamma}\,, \qquad D\bar\kappa_{\dot\alpha\dot\alpha} = \lambda^2 h^{\gamma}{}_{\dot\alpha} V_{\gamma\dot\alpha}\,.$$

Properties of the system (A)

1. AdS_4 covariant form

$$K_{AB} = K_{BA} = \begin{pmatrix} \lambda^{-1}\kappa_{\alpha\beta} & V_{\alpha\dot\beta} \\ V_{\beta\dot\alpha} & \lambda^{-1}\bar\kappa_{\dot\alpha\dot\beta} \end{pmatrix}, \qquad \Omega_{AB} = \Omega_{BA} = \begin{pmatrix} \Omega_{\alpha\beta} & -\lambda h_{\alpha\dot\beta} \\ -\lambda h_{\beta\dot\alpha} & \bar\Omega_{\dot\alpha\dot\beta} \end{pmatrix} =$$

$$D_0 K_{AB} = 0\,, \qquad D_0^2 \sim R_{0AB} = d\Omega_{AB} + \frac{1}{2}\Omega_A{}^C \wedge \Omega_{CB} = 0\,.$$

K_{AB} – AdS_4 **global symmetry parameter**

2. The existence of source-free Maxwell tensor

$$F_{\alpha\alpha} = -\lambda^{-2} G^3 \kappa_{\alpha\alpha}\,, \qquad \bar{F}_{\dot\alpha\dot\alpha} = -\lambda^{-2} \bar{G}^3 \bar\kappa_{\dot\alpha\dot\alpha}\,,$$

where

$$G = \frac{\lambda^2}{\sqrt{-\kappa^2}} = (-F^2)^{1/4}, \qquad \bar{G} = \frac{\lambda^2}{\sqrt{-\bar\kappa^2}} = (-\bar{F}^2)^{1/4}$$

$F_{\alpha\alpha}$ and $\overline{F}_{\dot\alpha\dot\alpha}$ satisfy source free Maxwell equations and Bianchi identities

$$D_{\gamma\dot\alpha} F_\alpha{}^\gamma = 0\,, \qquad D_{\alpha\dot\gamma} \bar{F}_{\dot\alpha}{}^{\dot\gamma} = 0.$$

AdS_4 Unfolded equations in terms of Maxwell field take the form:

$$DV_{\alpha\dot\alpha} = \frac{1}{2}\rho\, h^\gamma{}_{\dot\alpha} F_{\gamma\alpha} + \frac{1}{2}\bar\rho\, h_\alpha{}^{\dot\gamma} \bar{F}_{\dot\alpha\dot\gamma}\,,$$

$$DF_{\alpha\alpha} = -\frac{3}{2G} h^{\beta\dot\gamma} V^\beta{}_{\dot\gamma} F_{(\beta\beta} F_{\alpha\alpha)}\,, \quad D\bar{F}_{\dot\alpha\dot\alpha} = -\frac{3}{2} h^{\gamma\dot\beta} V_\gamma{}^{\dot\beta} \bar{F}_{(\dot\beta\dot\beta} \bar{F}_{\dot\alpha\dot\alpha)}\,.$$

with

$$\rho = -\lambda^2 G^{-3}, \qquad \bar\rho = -\lambda^2 \bar{G}^{-3}.$$

3. Two first integrals

$$I_1 = V^2 - \frac{\lambda^2}{2}\left(\frac{1}{G^2} + \frac{1}{\bar{G}^2}\right) ,$$

$$I_2 = \frac{1}{G^3\bar{G}^3} V^{\alpha\dot{\alpha}} V^{\alpha\dot{\alpha}} F_{\alpha\alpha} \bar{F}_{\dot{\alpha}\dot{\alpha}} - V^2 \left(\frac{1}{G^2} + \frac{1}{\bar{G}^2}\right) + \frac{\lambda^2}{4}\left(\frac{1}{G^2} - \frac{1}{\bar{G}^2}\right)^2 ,$$

$$dI_{1,2} = 0$$

related to two AdS_4 invariants (Casimir operators)

$$C_2 = \frac{1}{4} K_{AB} K^{AB} = I_1 , \qquad C_4 = \frac{1}{4} \mathrm{Tr} K^4 = I_1^2 + \lambda^2 I_2$$

4. The existence of Kerr-Schild vectors

Introduce projectors

$$\Pi^{\pm}_{\alpha\beta} = \frac{1}{2}(\epsilon_{\alpha\beta} \pm \frac{1}{G^2} F_{\alpha\beta}), \qquad \bar{\Pi}^{\pm}_{\dot{\alpha}\dot{\beta}} = \frac{1}{2}(\epsilon_{\dot{\alpha}\dot{\beta}} \pm \frac{1}{\bar{G}^2} \bar{F}_{\dot{\alpha}\dot{\beta}}) \,.$$

The projectors allow one to build light-light vectors for any given vector $V_{\alpha\dot{\alpha}}$

real: $$\xi^{+}_{\alpha\dot{\alpha}} = \Pi^{+\,\beta}_{\alpha} \bar{\Pi}^{+\,\dot{\beta}}_{\dot{\alpha}} V_{\beta\dot{\beta}} \,, \quad \xi^{-}_{\alpha\dot{\alpha}} = \Pi^{-\,\beta}_{\alpha} \bar{\Pi}^{-\,\dot{\beta}}_{\dot{\alpha}} V_{\beta\dot{\beta}}$$

complex: $$\xi^{+-}_{\alpha\dot{\alpha}} = \Pi^{+\,\beta}_{\alpha} \bar{\Pi}^{-\,\dot{\beta}}_{\dot{\alpha}} V_{\beta\dot{\beta}} \,, \quad \xi^{-+}_{\alpha\dot{\alpha}} = \Pi^{-\,\beta}_{\alpha} \bar{\Pi}^{+\,\dot{\beta}}_{\dot{\alpha}} V_{\beta\dot{\beta}}$$

$$\xi_{\alpha\dot{\alpha}} \xi^{\alpha\dot{\alpha}} = 0$$

Kerr-Schild vectors:

$$k_{\alpha\dot{\alpha}} = \frac{2}{(V^+V^-)} V^{-}_{\alpha\dot{\alpha}} \,, \qquad n_{\alpha\dot{\alpha}} = \frac{2}{(V^+V^-)} V^{+}_{\alpha\dot{\alpha}}$$

$$l^{-+}_{\alpha\dot{\alpha}} = \frac{2}{(V^{+-}V^{-+})} V^{-+}_{\alpha\dot{\alpha}} \,, \quad l^{+-}_{\alpha\dot{\alpha}} = \frac{2}{(V^{+-}V^{-+})} V^{+-}_{\alpha\dot{\alpha}}$$

Kerr-Schild relations

$$e_{I,\alpha\dot{\alpha}} = (k_{\alpha\dot{\alpha}}, n_{\alpha\dot{\alpha}}, l^{+-}_{\alpha\dot{\alpha}}, l^{-+}_{\alpha\dot{\alpha}})$$

$$e_{I,\alpha\dot{\alpha}} e_{I,}{}^{\alpha\dot{\alpha}} = 0\,, \qquad \frac{1}{2} e_{I,\alpha\dot{\alpha}} V^{\alpha\dot{\alpha}} = 1\,,$$

$$e_I^{\alpha\dot{\alpha}} D_{\alpha\dot{\alpha}} e_{I,\beta\dot{\beta}} = 0 \quad \text{(no summation over } I\text{)}$$

Towards black hole

1. Black hole Weyl tensor is of D-Petrov type i.e., $\mathbf{C}_{\alpha(4)} \sim \mathbf{F}_{\alpha\alpha}\mathbf{F}_{\alpha\alpha}$

2. Has at least two Killing vectors

3. Metric admits Kerr-Schild vector

Deformation of $AdS_4 \to$ black hole unfolded system (B)

(Keep the same form of the unfolded equations)

$$\mathcal{D}\mathcal{V}_{\alpha\dot\alpha} = \frac{1}{2}\rho\,\mathbf{h}^{\gamma}{}_{\dot\alpha}\mathcal{F}_{\gamma\alpha} + \frac{1}{2}\bar\rho\,\mathbf{h}_{\alpha}{}^{\dot\gamma}\bar{\mathcal{F}}_{\dot\alpha\dot\gamma}\,,$$

$$\mathcal{D}\mathcal{F}_{\alpha\alpha} = -\frac{3}{2\mathcal{G}}\mathbf{h}^{\beta\dot\gamma}\mathcal{V}^{\beta}{}_{\dot\gamma}\mathcal{F}_{(\beta\beta}\mathcal{F}_{\alpha\alpha)}\,,$$

$$\mathcal{D}\bar{\mathcal{F}}_{\dot\alpha\dot\alpha} = -\frac{3}{2\bar{\mathcal{G}}}\mathbf{h}^{\gamma\dot\beta}\mathcal{V}_{\gamma}{}^{\dot\beta}\bar{\mathcal{F}}_{(\dot\beta\dot\beta}\bar{\mathcal{F}}_{\dot\alpha\dot\alpha)}$$

Unlike the AdS_4 case with $\rho = -\lambda^2\mathcal{G}^{-3}$ we assume ρ to be arbitrary $\rho = \rho(\mathcal{G},\bar{\mathcal{G}})$

Bianchi identities: $\mathcal{D}^2 \sim \mathbf{R}\,, \qquad \mathcal{D}\mathbf{R} = 0$

fix $\rho(\mathcal{G},\bar{\mathcal{G}})$ uniquely in the form

$$\rho(\mathcal{G},\bar{\mathcal{G}}) = \mathcal{M} - \lambda^2\mathcal{G}^{-3} - \mathbf{q}\,\bar{\mathcal{G}}$$

Curvature 2-form is given by

$$\mathcal{R}_{\alpha\alpha} = \frac{\lambda^2}{2}\mathbf{H}_{\alpha\alpha} - \frac{3(\mathcal{M} - \mathbf{q}\,\bar{\mathcal{G}})}{4\mathcal{G}}\mathbf{H}^{\beta\beta}\mathcal{F}_{(\beta\beta}\mathcal{F}_{\alpha\alpha)} + \frac{\mathbf{q}}{4}\,\bar{\mathbf{H}}^{\dot\beta\dot\beta}\bar{\mathcal{F}}_{\dot\beta\dot\beta}\mathcal{F}_{\alpha\alpha}, \qquad \mathbf{H}_{\alpha\alpha} = h_{\alpha}{}^{\dot\alpha}\wedge h_{\alpha\dot\alpha}$$

AdS_4**-Kerr-Newman-Taub-NUT black hole (rotated, EM and NUT-charged)**

$\mathbf{M}$**=Re**$\mathcal{M}$ – black hole mass

$\mathbf{N}$**=Im**$\mathcal{M}$– NUT charge

$\mathbf{q} = e^2 + \mathrm{g}^2$ – sum of squared electric and magnetic charges

Properties of the system (B)

1. **Two integrals of motion**

$$\mathcal{I}_1 = \mathcal{V}^2 - \mathcal{M}\mathcal{G} - \overline{\mathcal{M}}\bar{\mathcal{G}} - \frac{\lambda^2}{2}\left(\frac{1}{\mathcal{G}^2} + \frac{1}{\bar{\mathcal{G}}^2}\right) + \mathbf{q}\,\mathcal{G}\bar{\mathcal{G}}\,,$$

$$\mathcal{I}_2 = \frac{1}{\mathcal{G}^3\bar{\mathcal{G}}^3}\mathcal{V}^{\alpha\dot\alpha}\mathcal{V}^{\alpha\dot\alpha}\mathcal{F}_{\alpha\alpha}\bar{\mathcal{F}}_{\dot\alpha\dot\alpha} - 2\left(\frac{\mathcal{M}}{\mathcal{G}} + \frac{\overline{\mathcal{M}}}{\bar{\mathcal{G}}}\right) - \mathcal{I}_1\left(\frac{1}{\mathcal{G}^2} + \frac{1}{\bar{\mathcal{G}}^2}\right) - \frac{\lambda^2}{4}\left(\frac{1}{\mathcal{G}^4} + \frac{1}{\bar{\mathcal{G}}^4}\right) - \frac{3\lambda^2}{2\mathcal{G}^2\bar{\mathcal{G}}^2}$$

2. $\mathcal{F}_{\alpha\alpha}$, $\bar{\mathcal{F}}_{\dot\alpha\dot\alpha}$ **form source free Maxwell tensor**

$$\mathcal{D}_{\alpha\dot\gamma}\bar{\mathcal{F}}_{\dot\alpha}{}^{\dot\gamma} = 0\,, \quad \mathcal{D}_{\gamma\dot\alpha}\mathcal{F}^{\alpha\gamma} = 0$$

3. **admits two real and two complex Kerr-Schild vectors**

real: $$k_{\alpha\dot\alpha} = \frac{2}{(V^+V^-)}V^-_{\alpha\dot\alpha}\,, \qquad n_{\alpha\dot\alpha} = \frac{2}{(V^+V^-)}V^+_{\alpha\dot\alpha}$$

complex: $$l^{-+}_{\alpha\dot\alpha} = \frac{2}{(V^{+-}V^{-+})}V^{-+}_{\alpha\dot\alpha}\,, \qquad l^{+-}_{\alpha\dot\alpha} = \frac{2}{(V^{+-}V^{-+})}V^{+-}_{\alpha\dot\alpha}$$

Systems (A) and (B) are algebraically similar in many respects and coincide at $\mathcal{M} = \mathrm{q} = 0$ describing empty AdS_4 space-time $\Rightarrow$ map to $\mathcal{M}, \mathrm{q} \neq 0$??

Integrating flow (A)⇔(B)

Let the deformation parameters $\chi = (\mathcal{M}, \bar{\mathcal{M}}, \mathbf{q})$ run $\Rightarrow$ one has corresponding flows $\frac{\partial}{\partial\chi}$. **Applying the integrability conditions to (B):**

$$[d, \frac{\partial}{\partial\chi}] = [\frac{\partial}{\partial\chi}, \frac{\partial}{\partial\chi'}] = 0$$

Example:

$$\partial_{\mathcal{M}} \mathcal{V}_{\alpha\dot\alpha} = \sum_{I=1}^{4} \phi_I \hat{e}_{I,\alpha\dot\alpha}\,, \qquad \partial_{\mathcal{M}} \mathbf{h}_{\alpha\dot\alpha} = \frac{1}{2}\sum_{I=1}^{4} \phi_I \hat{e}_{I,\alpha\dot\alpha} \hat{e}_{I,\beta\dot\beta} \mathbf{h}^{\beta\dot\beta}\,, \qquad \partial_{\chi} \mathcal{F}_{\alpha\alpha} = 0\,,$$

where

$$\phi_1 = \frac{\mathcal{G} + \bar{\mathcal{G}}}{4}\alpha_1(r)\,, \qquad \phi_2 = \frac{\mathcal{G} + \bar{\mathcal{G}}}{4}\alpha_2(r)\,,$$

$$\phi_3 = \frac{\mathcal{G} - \bar{\mathcal{G}}}{4}\beta_1(y)\,, \qquad \phi_4 = \frac{\mathcal{G} - \bar{\mathcal{G}}}{4}\beta_2(y)$$

and

$$r = \mathbf{Re}\frac{1}{\mathcal{G}}\,, \qquad y = \mathbf{Im}\frac{1}{G}\,, \qquad \alpha_1 + \alpha_2 = \beta_1 + \beta_2 = 1$$

Integration

Kerr-Schild vectors:

$$\widehat{k}_{\alpha\dot{\alpha}} = k_{\alpha\dot{\alpha}} \left(\frac{\Delta_r}{\widehat{\Delta}_r}\right)^{\alpha_2}, \quad \widehat{n}_{\alpha\dot{\alpha}} = n_{\alpha\dot{\alpha}} \left(\frac{\Delta_r}{\widehat{\Delta}_r}\right)^{\alpha_1},$$

$$\widehat{l}^{+-}_{\alpha\dot{\alpha}} = l^{+-}_{\alpha\dot{\alpha}} \left(\frac{\Delta_y}{\widehat{\Delta}_y}\right)^{\beta_2}, \quad \widehat{l}^{-+}_{\alpha\dot{\alpha}} = l^{-+}_{\alpha\dot{\alpha}} \left(\frac{\Delta_y}{\widehat{\Delta}_y}\right)^{\beta_1},$$

where

$$\widehat{\Delta}_r = 2\mathrm{M}r + r^2(\lambda^2 r^2 + \mathcal{I}_1) + \frac{1}{2}(-\mathbf{q} + \frac{\mathcal{I}_2}{2})$$

$$\widehat{\Delta}_y = 2\mathrm{N}y + y^2(\lambda^2 y^2 - \mathcal{I}_1) + \frac{1}{2}(\mathbf{q} + \frac{\mathcal{I}_2}{2}),$$

and

$$\Delta_{r,y} = \widehat{\Delta}_{r,y}\Big|_{\mathcal{M},\overline{\mathcal{M}},\mathbf{q}=0}, \qquad e_{I,\alpha\dot{\alpha}} = \widehat{e}_{I,\alpha\dot{\alpha}}\Big|_{\mathcal{M},\overline{\mathcal{M}},\mathbf{q}=0}.$$

Black hole metrics

- **General case** (Carter-Plebanski)
  deformation parameters: $\mathbf{M}$ – black hole mass, $\mathbf{N}$ – NUT charge, $\mathbf{q}$ – EM charges
  first integrals parameters: ϵ – Carter constant, **a** – angular momentum

$$ds^2 = ds_0^2 + \frac{2\mathrm{M}r - \mathbf{q}/2}{r^2+y^2}(\alpha_1(r)K + \alpha_2(r)N)^2 - \frac{2\mathrm{N}y + \mathbf{q}/2}{r^2+y^2}(\beta_1(y)L^{+-} + \beta_2(y)L^{-+})^2$$

$$+4\alpha_1(r)\alpha_2(r)\frac{r^2+y^2}{\Delta_r\widehat{\Delta}_r}(2\mathrm{M}r - \mathbf{q}/2)dr^2 - 4\beta_1(y)\beta_2(y)\frac{r^2+y^2}{\Delta_y\widehat{\Delta}_y}(2\mathrm{N}y + \mathbf{q}/2)dy^2,$$

where

$$K = k_\mu dx^\mu\,, \quad N = n_\mu dx^\mu\,, \quad L^{+-} = l_\mu^{+-}dx^\mu\,, \quad L^{-+} = l_\mu^{-+}dx^\mu$$

and kinematical parameters ϵ, a are expressed via AdS_4 Casimir invariants

$$\epsilon = C_2\,, \quad 4\lambda^2 a^2 = C_4 - C_2^2$$

- **Kerr-Newman case** (rotated and charged black hole)

$$ds^2 = ds_0^2 + \frac{2\mathrm{M}r - \frac{\mathrm{q}}{2}}{r^2 + y^2} k_\mu k_\nu dx^\mu dx^\nu \,.$$

$$C_2 = 1 + \lambda^2 a^2 \,, \qquad C_4 = C_2^2 + 4\lambda^2 a^2 \,.$$

- **Reissner-Nordström** (static charged black hole)

$$C_4 = C_2^2 \neq 0 \,, \quad (K_A{}^C K_C{}^B \sim \delta_A{}^B)$$

Conclusion

- It is shown that a wide class of black hole metrics (Carter-Plebanski) admits simple unfolded description in terms of Killing and source-free Maxwell fields. The system is obtained as a parametric deformation of AdS_4 global symmetry equation. Two deformation parameters $\mathcal{M} \in \mathbb{C}$ and $\mathbf{q} \in \mathbb{R}$ are associated with black hole mass $\mathbf{M}$=Re $\mathcal{M}$, NUT charge $\mathbf{N}$=Im $\mathcal{M}$ and electro-magnetic charges $\mathbf{q} = e^2+\mathrm{g}^2$. Black hole kinematic characteristics related to the angular momentum a and the Carter parameter ϵ are expressed via two first integrals of the unfolded system

- Type of a black hole whether it is rotated or static is defined by the values of AdS_4 invariants (Casimir operators). In particular static black hole is defined by

$$C_4 = C_2^2$$

- The proposed formulation gives rise to a coordinate independent description of the black hole metric in AdS_4

- Black hole Fronsdal fields result as a simple consequence in the unfolded system