Ground Ring of α-Generators and Sequence of RNS String Theories

SFT 09

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- The purpose of this talk is to point out the existence of new local gauge symmetries in RNS superstring theory, leading to an infinite chain of new nilpotent BRST generators that can be classified in terms of ghost cohomologies

- These gauge symmetries are closely related to global nonlinear space-time α-symmetries in RNS superstring theory that mix matter and ghost degrees of freedom, form ground rings and originate from hidden space-time dimensions.

- The new nilpotent BRST charges, which construction will be demonstrated in this talk, correspond to sequence of RNS superstring theories in curved backgrounds (including AdS-type) and can be used to develop SFT's around nontrivial backgrounds

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In terms of RNS - Pure Spinor (PS) correspondence, we show that the appearance of new BRST charges corresponds to introducing interactions for the pure spinor variable λ^α in the PS BRST operator $\oint \frac{dz}{2i\pi}\lambda^\alpha d_\alpha$ (equivalent to OPE singularities OPE between λ's that preserve the nilpotence of Q_{BRST}). The orders of ghost cohomologies of BRST charges in RNS formalism correspond to the leading order of OPE singularity of two pure spinors in PS formalism.

- In string theory the global space-time symmetries are typically generated by primary fields of conformal dimension 1 (commuting with BRST charge), while local gauge symmetry are given by BRST exact operators (of various conformal dimensions and not necessarily primary) , given by commutators of BRST operator with appropriate ghost fields. ields.

- Examples of generators of local gauge symmetries on the worldsheet are the stress-energy tensor T and the supercurrent G: $T = \{Q_0, b\}$ and $G = [Q_0, \beta]$, where

$$Q_0 = \oint \frac{dz}{2i\pi}(cT + \partial ccb - \frac{1}{2}\gamma\psi_m\partial X^m - \frac{1}{4}b\gamma^2)$$

is the standard BRST charge while the dimension 1 primaries $L^m = \oint \frac{dz}{2i\pi}\partial X^m$ and $L^{mn} = \oint \frac{dz}{2i\pi}\psi^m\psi^n$ generate Lorenz translations and rotations on the worldsheet

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To construct a complete version of the generator of Lorenz rotations, which acts both on X's and ψ's one needs to improve L^{mn} with bc-ghost dependent terms, so the complete BRST-invariant expression for the rotation generator is given by

$$L^{\tilde{mn}} = L^{mn} - 2\oint \frac{dz}{2i\pi} c\xi e^{-\phi}\partial X^{[m}\psi^{n]} - \frac{1}{2}\partial c c e^{3\chi-3\phi}\partial X^{[m}\psi^{n]}$$

with the bosonic and fermionic ghosts β, γ, b, c bosonized as

$$\gamma(z) = e^{\phi-\chi}(z);$$
$$\beta(z) = e^{\chi-\phi}\partial\chi(z) \equiv \partial\xi e^{-\phi}(z)$$
$$b(z) = e^{-\sigma}(z); c(z) = e^{\sigma}(z)$$

This defines the BRST-invariant rotation generator, acting both on ψ's and (up to a picture-changing) on X's

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The next, far less trivial example of global space-time supersymmetry in superstring theory is given by the hierarchy of α-symmetries. These global space-time symmetries are realised non-linearly, mixing the matter and the ghost sectors of RNS superstring theory and can be classified in terms of ghost cohomologies. Namely, it can be checked that the full matter+ghost RNS action:

$$\begin{aligned}
S_{RNS} &= S_{matter} + S_{bc} + S_{\beta\gamma} \\
S_{matter} &= \frac{1}{2\pi}\int d^2z(\partial X_m \bar{\partial} X^m \\
&\quad + \psi_m \bar{\partial}\psi^m + \bar{\psi}_m \partial \bar{\psi}^m) \\
S_{bc} &= \frac{1}{2\pi}\int d^2z(b\bar{\partial}c + \bar{b}\partial\bar{c}) \\
S_{\beta\gamma} &= \frac{1}{2\pi}\int d^2z(\beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma})
\end{aligned}$$

is invariant under the following transformations (with α being a global parameter):

$$\delta X^m = \alpha\{2e^{\phi}\partial\psi^m + \partial(e^{\phi}\psi^m)\}$$
$$\delta\psi^m = -\alpha\{e^{\phi}\partial^2 X^m + 2\partial(e^{\phi}\partial X^m)\}$$
$$\delta\gamma = \alpha e^{2\phi-\chi}\{\psi_m\partial^2 X^m - 2\partial\psi_m\partial X^m\}$$
$$\delta\beta = \delta b = \delta c = 0$$

so that

$$\delta S_{matter} = -\delta S_{\beta\gamma}$$
$$= \frac{1}{2\pi}\int d^2z(\bar{\partial}e^{\phi})(\psi_m\partial^2 X^m - 2\partial\psi_m\partial X^m)$$
$$\delta S_{bc} = \delta S_{RNS} = 0$$

The generator of these transformations is given by

$$L^{\alpha} = \oint \frac{dz}{2i\pi} e^{\phi} F(X,\psi)$$

$$\equiv \oint \frac{dz}{2i\pi} e^{\phi} (\psi_m \partial^2 X^m - 2\partial\psi_m \partial X^m)$$

where it is convenient to introduce the notation for the dimension $\frac{5}{2}$ primary field:

$$F(X,\psi) = \psi_m \partial^2 X^m - 2\partial\psi_m \partial X^m$$

along with the matter worldsheet supercurrent

$$G = -\frac{1}{2}\psi_m \partial X^m$$

and the dimension 2 primary

$$L(X,\psi) = 2\partial\psi_m\psi^m - \partial X_m \partial X^m$$

which is the w.s. superpartner of F, i.e.

$$G(z)L(w) \sim \frac{F(w)}{z-w}$$

L^α-generator is the element of the ghost cohomology H_1. As in the case of the rotation generator, the integrand of the L^α -generator is a primary field of dimension 1, however it is not BRST-invariant since it doesn't commute with the supercurrent terms of the BRST charge; so similarly one has to introduce the bc-dependent correction terms to make it BRST-invariant.

Definition

- Positive ghost cohomologies H_n ($n > 0$) consist of picture-inequivalent physical operators, existing at pictures n and above, annihilated by inverse picture changing transformation at minimal positive picture n.

- Negative ghost cohomologies H_{-n} consist of picture-ineguivalent physical operators, existing at pictures $-n$ and below, annihilated by direct picture changing at minimal negative picture $-n$.

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An isomorphism holds between positive and negative cohomologies:

$$H_n \sim H_{-n-2}$$

H_0 by definition consists of picture-equivalent operators existing at all pictures (including picture 0), while H_{-1} and H_{-2} are empty.

The full BRST-invariant extension of L^α generating the complete set of α-symmetries for the matter and the ghost sectors is given by:

$$\tilde{L}^\alpha(w) = \oint \frac{dz}{2i\pi}(z-w)^2\{e^\phi F P^{(2)}_{2\phi-2\chi-\sigma}(z)$$
$$+8c\xi(FG - \frac{1}{2}LP^{(2)}_{\phi-\chi} - \frac{1}{4}\partial L P^{(1)}_{\phi-\chi})$$
$$-24\partial c c e^{2\chi-\phi}F\} \equiv \oint \frac{dz}{2i\pi}(z-w)^2 V_3$$

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where the conformal weight n polynomials $P^{(n)}_{f(\phi_1(z),...,\phi_N(z))}$ are the generalized Hermite polynomials defined as

$$P^{(n)}_{f(\phi_1(z),...,\phi_N(z))}$$

$$= e^{-f(\phi_1(z),...,\phi_N(z))}\frac{\partial^n}{\partial z^n}e^{f(\phi_1(z),...,\phi_N(z))}$$

for an arbitrary function f of N fields $\phi_1(z), ..., \phi_N(z)$ (e.g. $P^{(1)}_{2\phi-2\chi-\sigma} = 2\partial\phi - 2\partial\chi - \partial\sigma$)

The operator $\tilde{L}^\alpha$ is BRST-invariant and non-trivial, generating the full set of global nonlinear space-time symmetries, originating from hidden dimensions. Note that the dimension 3 integrand of $\tilde{L}^\alpha$ satisfies

$$[Q_0, V_3] = \partial^3 W_0 \qquad (1)$$

where W_0 is dimension 0 operator (which precise form is skipped for brevity)

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While it depends on an arbitrary worldsheet coordinate w, this dependence doesn't affect any correlation functions, as the w derivatives of $\tilde{L}^{\alpha}(w)$ are BRST exact, forming the ground ring.

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The non-vanishing operators are the first and the second derivatives of $\tilde{L}^{\alpha}(w)$ in w, given by

$$L_1^{\alpha}(w) = \partial_w L^{\alpha}(w)$$

$$= -2\oint \frac{dz}{2i\pi}(z-w)\{e^{\phi} F P^{(2)}_{2\phi-2\chi-\sigma}(z)$$

$$+8c\xi(FG - \frac{1}{2}LP^{(2)}_{\phi-\chi} - \frac{1}{4}\partial L P^{(1)}_{\phi-\chi})$$

$$-24\partial c c e^{2\chi-\phi} F\}$$

and

$$L_2^{\alpha}(w) = \partial_w L_1^{\alpha}(w)$$

It is straighforward to show that these generators induce local gauge symmetries on the worldsheet and are BRST exact,that is :

$$L_1^{\alpha}(w) = \{Q_0, \oint \frac{dz}{2i\pi}\tilde{b}(z,w)\}$$
$$L_2^{\alpha}(w) = \{Q_0, \partial_w \oint \frac{dz}{2i\pi}\tilde{b}(z,w)\} \quad (2)$$

with the role of the generalized b-ghost (corresponding to gauge transformations induced by $L_j^\alpha \equiv \partial_w^j L^\alpha(w); j = 1,2$) played by

$$\oint \frac{dz}{2i\pi}\tilde{b}(z,w)$$

$$= \oint \frac{dz}{2i\pi}(z-w)^2\{-2be^{\phi}FP^{(1)}_{2\phi-2\chi-\sigma}(z)$$

$$+8\xi(FG - \frac{1}{2}LP^{(2)}_{\phi-\chi} - \frac{1}{4}\partial LP^{(1)}_{\phi-\chi})$$

$$+24\partial ce^{2\chi-\phi}F\}$$

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Note: the integrands of L_1^α and $\tilde{b}$ are conformal dimension 2 generators.

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Now that we have the $\tilde{b}$-analogue of the b-ghost, what is the generalized $\tilde{c}$ ghost?

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In analogy with the usual c-ghost, we shall look for conformal dimension -1 operator, satisfying the canonical relation

$$\{\oint \tilde{b}, \tilde{c}\} = 1 \qquad (3)$$

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Complication: since $\tilde{b}$ is at picture $+1$, $\tilde{c}$ must be at picture -1 to satisfy (). It appears there is no suitable expression for $\tilde{c}$ satisfying these conditions. However, since $\tilde{L}_\alpha$ is on-shell, the picture-changing transformation is applicable to it.

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Since

$$L_1^\alpha = \{Q_0, \oint \tilde{b}\}$$

and picture changing operators Γ and Γ^{-1} (direct and inverse) are BRST-invariant, one has

$$\Gamma^n L_1^\alpha = \{Q_0, \Gamma^n \oint \tilde{b}\}$$

so the p.c. transform can be applied to generalized ghosts as well (even though they are off-shell).

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It is convenient to bring $\oint \tilde{b}$ to picture -1 by applying Γ^{-1} twice. The picture -1 expression for $\oint \tilde{b}$ is given by:

$$\oint \tilde{b}(w) = \oint \frac{dz}{2i\pi}\{-8\partial cce^{3\phi-4\chi} \times$$

$$\{\frac{1}{2}P^{(2)}_{-\sigma}[-\frac{3}{8}\partial^2 L + \frac{1}{4}\partial L P^{(1)}_{-16\phi+3\chi-3\sigma} + L$$

$$\times(-\frac{3}{2}\partial^2\phi + \frac{11}{8}\partial^2\chi + \frac{3}{8}\partial^2\sigma - 4(\partial\phi)^2 + \frac{5}{8}(\partial\chi)^2$$

$$+\frac{1}{8}(\partial\sigma)^2 + 6\partial\phi\partial\chi - \frac{1}{2}\partial\phi\partial\sigma + \frac{7}{4}\partial\chi\partial\sigma)]$$

$$+\frac{1}{6}P^{(3)}_{-\sigma}(-\frac{3}{4}\partial L + L(\frac{1}{4}\partial\sigma - \frac{1}{2}\partial\phi)) + \frac{1}{48}P^{(4)}_{-\sigma}L\}$$

$$-ce^{2\chi-3\phi}\{P^{(1)}_{-\sigma} \times [-\frac{3}{8}\partial^2 F - \frac{1}{4}\partial F P^{(1)}_{\phi-2\chi+2\sigma}$$

$$+F[\frac{1}{8}\partial^2\phi + \frac{15}{4}\partial^2\chi - \frac{1}{4}\partial^2\sigma + \frac{13}{8}(\partial\phi)^2 =$$

$$-3(\partial\chi)^2 - \frac{5}{2}\partial\phi\partial\chi - \frac{3}{2}\partial\phi\partial\sigma + \partial\chi\partial\sigma]$$

$$+\frac{1}{2}P^{(2)}_{-\sigma}(\frac{-1}{2}\partial F + F(-\frac{3}{2}\partial\phi - \partial\chi)) - \frac{1}{24}P^{(3)}_{-\sigma}\}$$

Next, the conjugate $\tilde{c}$-ghost, satisfying

$$\{\oint \tilde{b}, \tilde{c}\} = \Gamma$$

(note that Γ is picture-changing operator, i.e. picture-transformed unit operator 1) is given by:

$$
\begin{aligned}
\tilde{c} = \frac{1}{2}e^{3\phi-\chi}\{F(\frac{1}{3}P^{(3)}_{\phi-\chi} + \frac{1}{2}\partial\phi P^{(2)}_{\phi-\chi}) \\
+GL(\frac{1}{2}P^{(2)}_{\phi-\chi} + \partial\phi P^{(1)}_{\phi-\chi} + \frac{1}{2}\partial F P^{(2)}_{\phi-\chi}) \\
+\partial GLP^{(1)}_{2\phi-\chi} + G\partial LP^{(1)}_{\phi-\chi} \\
+\frac{1}{2}\partial^2 GL + \partial G\partial L\} \\
+be^{4\phi-2\chi}\{\frac{1}{2}GFP^{(1)}_{\phi-\chi-\frac{3}{4}\sigma}P^{(1)}_{\phi-\chi} \\
+\frac{1}{12}LP^{(3)}_{\phi-\chi}P^{(1)}_{\phi-\chi-\frac{3}{4}\sigma} \\
+\frac{1}{16}\partial LP^{(2)}_{\phi-\chi}P^{(1)}_{\phi-\chi-\frac{3}{4}\sigma}\} \\
+\partial bbe^{5\phi-3\chi}\{-\frac{1}{8}P^{(1)}_{\phi-\chi-\frac{3}{4}\sigma}P^{(2)}_{2\phi-2\chi-\sigma} \\
+\frac{1}{32}P^{(3)}_{2\phi-2\chi-\sigma}\}
\end{aligned}
$$

Since the ground ring elements L_1^α and

L_2^α can be shown to commute:

$$[L_1^\alpha, L_2^\alpha] = 0$$

the nilpotent BRST charge of ghost cohomology H_1 is by definition equal to

$$Q_1 = \tilde{c}_1 L_1^\alpha + \tilde{c}_2 L_2^\alpha$$
$$\tilde{c}_1 \equiv \tilde{c}, \tilde{c}_2 = \oint \tilde{c}$$

Computing the OPE's it is straightforward to derive the manifest integrated expression for Q_1:

$$Q_1 = \oint \frac{dz}{2i\pi}\{ce^\phi(GL + FP^{(1)}_{\phi-\chi})$$
$$+\frac{1}{4}e^{2\phi-\chi}(GF + \frac{1}{2}LP^{(2)}_{2\phi-2\chi-\sigma}) - \partial cc\xi L(z)\}$$

This defines a new BRST complex in RNS superstring theory! It is an element of superconformal ghost cohomology H_1

BRST charges of Higher Order BRST Cohomologies

In case of uncompactified critical RNS superstring theory the α-symmetry () is the only additional global space-time symmetry, present in the theory. For RNS theories in noncritical dimensions or critical but compactified on S^1, there is a huge set of additional α-symmetries, due to interactions with the Liouville (or compactified) mode.

(D.P. 0706.0275, IJMPA (2007); 0806.3565, IJMPA (2009))

Thus, for a d-dimensional RNS theory, there exist $d+1$ additional α-symmetries of ghost cohomology H_1. Combined with $\frac{(d+1)(d+2)}{2}$ Poincare symmetries (including the Liouville direction), these d+2 ghost-matter mixing symmetries of H_1 enlarge space-time symmetry group from $SO(2,d)$ to $SO(2,d+1)$, bringing in the first extra-dimension. Next , an H_2 cohomology can be shown to contain $(d+3)$ superconformal ghost number 2 α-symmetries which, combined with Poincare symmetries and α-symmetries of H_2 anrarge the space-time symmetry group to $SO(2,d+2)$, bringing in the second extra-dimension.

Example of a typical α-generator of H_2:

$$L^{\beta} = \oint \frac{dz}{2i\pi} e^{2\phi} F(X,\psi) F(\varphi,\lambda)(z)$$
$$F(X,\psi) = \psi_m \partial^2 X^m - 2\partial\psi_m \partial X^m$$
$$F(\varphi,\lambda) = \lambda\partial^2\varphi - \partial\lambda\partial\varphi$$

where ϕ and λ are the super Liouville components (or those of a compactified direction)

As in the case of L^{α}, BRST-symmetrization of

$$L^{\beta} \to \tilde{L}^{\beta}(w)$$

leads to correction terms which w-derivatives give rise to ground ring, inducing gauge transformations at the H_2-level with the associate ghost pair $\oint \tilde{b}^{(2)}(w)$ and $\oint \widetilde{c^{(2)}}(w)$

satisfying

$$L_1^\beta = \{Q_0, \oint \tilde{b}^{(2)}(w)\}$$
$$\{\oint \tilde{b}^{(2)}(w), \tilde{c}^{(2)}\} = \Gamma^2$$

At this level, however, the ground ring is non-commutative and consists of 3 elements: with

$$L_i^\beta \equiv \partial_w^i L^\beta(w); i = 1, 2, 3$$

satisfying

$$[L_1^\beta, L_2^\beta] = \frac{1}{2} L_3^\beta$$

So the BRST charge at the H_2-level is

$$Q_2 = \sum_{j=1}^{3} c\tilde{^{(2)}}_j L_j^\beta + \frac{1}{4} c^{(2)}{}_1 c^{(2)}{}_2 \partial_w^2 \oint \tilde{b}^{(2)}(w)$$

This construction can in principle be generalized to ghost cohomologies H_n of arbitrary n, with each cohomology rank having its own associate BRST charge Q_n; although I only was able to do it explicitly for $n \leq 3$.

Determining BRST cohomologies of Q_n for $n \geq 1$ is a challenging and interesting problem, aland though it looks plausible that each of Q_n corresponds to RNS string theory with certain background geometry.

Properties of Q_n: cohomologies

So far, we have been able to investigate the simplest nontrivial case $n = 1$. The problem of investigating the higher n cases is still to be addressed. In critical uncompactified case the only nontrivial element of Q_0+Q_1 is given by the massless gauge boson:

$$V(k) = \oint e^{-3\phi}\{(\vec{A}\partial\vec{X})(\vec{k}\partial\vec{X})(\vec{k}\vec{\psi})$$
$$+(\vec{A}\vec{\psi})(\vec{k}\partial\vec{\psi})(\vec{k}\vec{\psi})(\vec{A}\vec{\psi})(\vec{k}\partial\vec{X})^2\}e^{i\vec{k}\vec{X}}$$
$$\vec{k}\vec{A}(\vec{k}) = 0; (\vec{k})^2 = 0$$

This operator is the element of H_{-3}. In the noncritical cases there are other massless modes. in particular, for $d = 4$ there are 7 extra massless vector bosons in the $Q_0 + Q_1$ cohomology, altogether giving rise to SU(3) octet of gluons. **There**

are no nontrivial massive modes in the cohomology! Such a field-theoretic behaviour is characterictic for string theories in AdS-type backgrounds, dual to CFT (YM in $d = 4$)

D.P.,0806.3565;IJMPA24:113(2009)

New BRST Charges and Deformed Pure Spinors

In pure spinor formalism the standard BRST operator

$$
\begin{aligned}
Q_{PS} = \oint \frac{dz}{2i\pi} \lambda^\alpha d_\alpha \\
d_\alpha = p_\alpha - \frac{1}{2}\gamma^m_{\alpha\beta}\partial X_m \theta^\beta \\
-\frac{1}{8}(\theta\gamma^m\partial\theta)(\gamma_m\theta)_\alpha \\
\alpha, \beta = 1, ..., 16
\end{aligned}
$$

is nilpotent if

$$
\lambda\gamma^m\lambda = 0
$$

(pure spinor condition) and OPE between two λ's is nonsingular. The latter condition is ensured by the fact that in the standard PS action λ is a free ghost field. This condition, however, can be relaxed with Q still remaining nilpotent even if OPE between pure spinors becomes singular, provided that the pure spinor constraint is still satisfied at a normal ordered level and certain other constraints are fulfilled. E.g. consider the most general OPE between $d_\alpha(z)$ $d_\beta(w)$ around the midpoint:

$$d_\alpha(z)d_\beta(w) = -\frac{\gamma^m_{\alpha\beta}\Pi^{(1)}_m(\frac{z+w}{2})}{z-w}$$
$$+(z-w)^0\gamma^{m_1\ldots m_3}_{\alpha\beta}\Pi^{(2)}_{m_1\ldots m_3}(\frac{z+w}{2})$$
$$+(z-w)\{\alpha_1\gamma^m_{\alpha\beta}\Pi^{(3)}_m$$
$$+\alpha_2\gamma^{m_1\ldots m_5}\Pi^{(3)}_{m_1\ldots m_5}\}(\frac{z+w}{2}) \quad (4)$$

and suppose that λ satisfies the OPE

$$\lambda_\alpha(z)\lambda_\beta(w) \sim (z-w)^{-2}\gamma^m_{\alpha\beta}A_m(\frac{z+w}{2}) + O(z-w)$$

(no $(z-w)^0$-term means that the PS constraint is fulfilled in a normal ordered (weaker) sense). Then the BRST charge is still nilpotent if either $\alpha_1 = 0$ or $: A_m \Pi^{(3)}_m := 0$ (other singularities vanish upon evaluating traces of gamma-matrices). This precisely is the situation that is realised if one considers the RNS-PS map

$$\theta_\alpha = e^{\frac{1}{2}\phi}\Sigma_\alpha;$$

$$\lambda_\alpha = \{Q_0, \theta_\alpha\}$$

$$= -\frac{1}{4}be^{\frac{5}{2}\phi-2\chi}\Sigma_\alpha - \frac{1}{2}e^{\frac{3}{2}\phi-\chi}\gamma^m_{\alpha\beta}\partial X_m\tilde{\Sigma}^\beta$$

$$+ce^{\frac{1}{2}\phi}(\frac{1}{2}\Sigma_\alpha\partial\phi + \partial\Sigma_\alpha)$$

) so that

$$\lambda_\alpha\lambda_\beta \sim \frac{1}{(z-w)^2}\partial bbe^{5\phi-4\chi}\gamma^m_{\alpha\beta}\psi_m + O(z-w)$$

It can be shown that, under such a PS-RNS identification the PS BRST charge is simply mapped to RNS BRST charge Q_0 (up to similarity transformation) and is therefore nilpotent:

(D.P. 0810.4696, to appear in IJMPA)

$$Q^{PS} \to e^{-R} Q_0^{RNS} e^{R}$$

where

$$R = 32 \oint \frac{dz}{2i\pi} \partial c c e^{2\chi - 2\phi} \partial \chi(z)$$

Thus the RNS theory is equivalent to modified PS theory with the double pole singularity in the pure spinor OPE. This construction can be generalized to include the modified pure spinors with more singular OPE; remarkably, the RNS BRST operators of higher ghost cohomologies are then reproduced, with the leading singularity order of pure spinor OPE related to the ghost cohomology order in RNS formalism (!) We have been able to show this for the $n = 1$ case and conjectured for higher n's. Namely, in the

$n = 1$ case one starts with

$$\tilde{\theta}^{\alpha} = e^{\frac{3}{2}\phi}\Sigma^{\beta}\gamma^{m}_{\alpha\beta}(2\partial^2 X_m + \partial X_m \partial\phi)$$

which is the dimension 0 primary field space-time spinor at ghost number $\frac{3}{2}$, **NOT** related to the previous ghost number $\frac{1}{2}$ version of θ by picture-changing.

Next, one defines

$$\tilde{\lambda}_{\alpha} = \{Q_0^{RNS}, \tilde{\theta}_{\alpha}\}$$

keeping d^{α} unchanged at picture $-\frac{1}{2}$

The straightforward calculation gives

$$Q_1^{PS} = \oint \tilde{\lambda}_{\alpha} d^{\alpha} \to e^{-R} Q_1^{RNS} e^{R}$$

with the same R.

Conjecture:

RNS superstring theory with the BRST operator Q_n of H_n is equivalent to modified pure spinor (PS) theory with singular pure spinor OPE's with the leading OPE singularity order given by $6n^2 + 6n + 2$.

Conclusions

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Hierarchy of surprising space-time α-symmetries in RNS superstring theories induces ground rings of matter-ghost mixing local gauge symmetries that can be classified in terms of ghost cohomologies H_n.

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Each ground ring induces the associate new BRST charge Q_n of H_n in RNS theory, corresponding to some deformed background geometry (AdS-type for $n = 1$)

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Each Q_n of RNS theory corresponds to deformed PS superstring theory, with the leading OPE singularity order in the PS formalism related to the ghost cohomology order of n in RNS formalism

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Projects for the future: investigate cohomologies of Q_n for $n \geq 1$; identify geometries of underlying backgrounds, building SFT's around these backgrounds...

possibly developing PS-formulated SFT's inspired by generalized RNS-PS map